Lecture 16
14.6 Tangent planes
14.7 Extreme values and saddle points

Jeremiah Southwick

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Things to note

Upcoming dates:
Quiz 7 today, no quiz on Friday.
Monday: Quiz 8 and WF drop date (see grade calculation sheet on Blackboard)
Wednesday, March 6: Review
Friday, March 8: Exam 2

Last class

At (a, b), f increases most rapidly in the direction of $\nabla f(a, b)$. $(\theta = 0)$.

At (a, b), f decreases most rapidly in the opposite direction of $\nabla f(a, b)$. $(\theta = \pi)$.

At (a, b), f is neither rising nor dropping in the directions orthogonal to $\nabla f(a, b)$. $(\theta = \pi/2)$.

14.6 Tangent planes

Question

How do we find a plane that is tangent to a function $z = f(x, y)$?

First we need a normal vector $\vec{\mathbf{n}}$ and a point $P_0 = (a, b, c)$ in the plane. Then the equation is

$$\vec{\mathbf{n}} \cdot \langle x - a, y - b, z - c \rangle = 0.$$

Idea: First find some lines tangent to the function.

Start off with tangent lines in the x- and y-directions.

Tangent planes

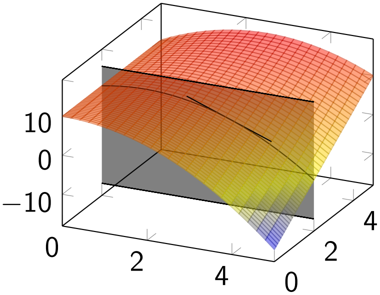
Question

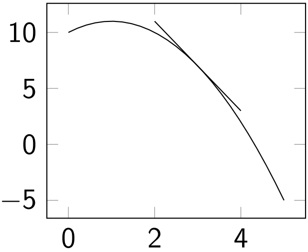
How do we find the slope of a function $z = f(x, y)$ *in the* x*-direction?*

Tangent planes

Question

How do we find the slope of a function $z = f(x, y)$ in the x-direction?





Tangent planes

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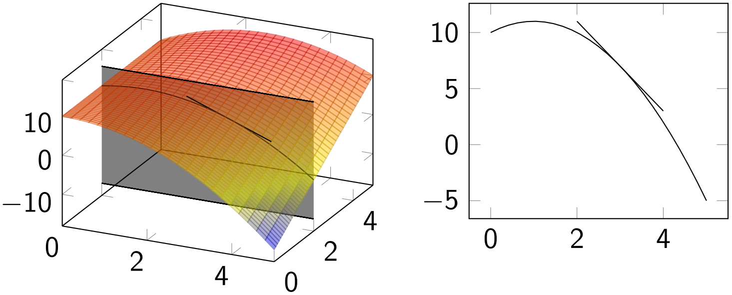
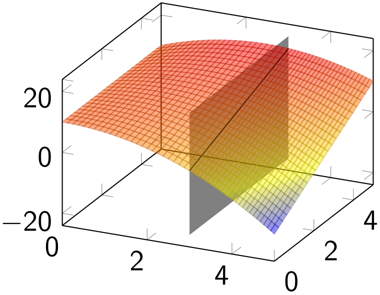


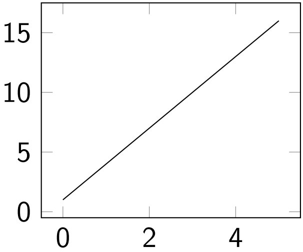
This line is given by $z - f(a,b) = f_x(a,b)(x-a)$ or

$$\vec{\mathbf{r}}(t) = \langle a, b, f(a,b) \rangle + t\langle 1, 0, f_x(a,b) \rangle.$$

Tangent planes

We can do the same thing in the y-direction.





Tangent planes

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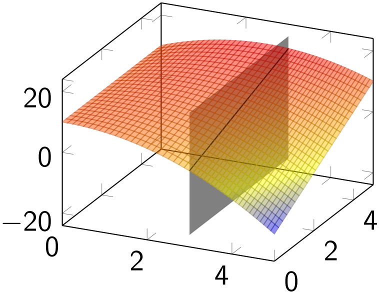

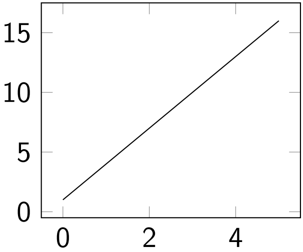

This line is given by $z - f(a, b) = f_y(a, b)(y - b)$ or

$$\vec{\mathbf{r}}(t) = \langle a, b, f(a, b)\rangle + t\langle 0, 1, f_y(a, b)\rangle.$$

Tangent planes

Question

How do we find a plane that is tangent to a function $z = f(x, y)$*?*

Tangent planes

Question

How do we find a plane that is tangent to a function $z = f(x, y)$?

Two tangent lines:

$$\vec{\mathbf{r}}(t) = \langle a, b, f(a, b)\rangle + t\langle 0, 1, f_y(a, b)\rangle$$

and

$$\vec{\mathbf{r}}(t) = \langle a, b, f(a, b)\rangle + t\langle 1, 0, f_x(a, b)\rangle.$$

Tangent planes

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Point: $P_0 = (a, b, f(a, b))$

Tangent planes

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Point: $P_0 = (a, b, f(a, b))$
Normal vector: $\vec{\mathbf{n}}$ is the cross product of the direction vectors of the lines.

Tangent planes

Point: $P_0 = (a, b, f(a, b))$

$$\vec{\mathbf{n}} =$$

Tangent planes

Point: $P_0 = (a, b, f(a,b))$

$$\vec{\mathbf{n}} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 0 & 1 & f_y(a,b) \\ 1 & 0 & f_x(a,b) \end{vmatrix} = \langle f_x(a,b), f_y(a,b), -1 \rangle$$

Tangent planes

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Then the equation of the plane is $\vec{\mathbf{n}} \cdot \langle x - a, y - b, z - f(a, b) \rangle = 0$ or

$$f_x(a, b)(x - a) + f_y(a, b)(y - b) - (z - f(a, b)) = 0.$$

Solving for z, we have

$$z = f_x(a, b)(x - a) + f_y(a, b)(y - b) + f(a, b).$$

Tangent plane example

$$z = f_x(a, b)(x - a) + f_y(a, b)(y - b) + f(a, b).$$

Example

Find the tangent plane to $z = x\cos(y) - ye^x$ *at* $(\ln(2), 0, \ln(2))$.

Tangent plane example

$$z = f_x(a,b)(x-a) + f_y(a,b)(y-b) + f(a,b).$$

Example

Find the tangent plane to $z = x\cos(y) - ye^x$ *at* $(\ln(2), 0, \ln(2))$.

We have $\nabla f = \langle \cos(y) - ye^x, -x\sin(y) - e^x \rangle$ and thus the equation is

$$(x - \ln(2)) + (-2)(y - 0) - (z - \ln(2)) = 0$$

or

$$z = x - 2y.$$